Nondegenerate Surface-Wave Mode Coupling Between Dielectric Waveguides

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Abstract—**A new variational derivation of the coupling coefficient is given for the problem of nondegenerate surface-wave modes on parallel dielectric waveguides. The results coincide with those of a number of different methods in the literature on the degenerate case, but all give distinct results in the nondegenerate case. These differences are examined and compared with the exact solution, whereby the approximations involved can be evaluated for the case of two parallel slab waveguides.**

I. INTRODUCTION

THERE EXISTS in the literature a number of theoretical treatments of the problem of coupling between parallel dielectric-waveguide surface-wave modes [1]–[7] for application to optical guiding structures. It is the purpose of this paper to derive by a variational technique a new expression for the coupling between two such guides, and to investigate the relationships between the various theories in situations where their results differ, that is, in the nondegenerate case. For the case of two parallel slab waveguides these results are compared with a numerical solution of the exact modal equation, and conditions for the validity of the various approximations are given.

II. COUPLING BETWEEN TWO ISOTROPIC GUIDES

For simplicity we consider first the case of two parallel isotropic (but possibly lossy and inhomogeneous) waveguides as shown in Fig. 1. The relative permittivities

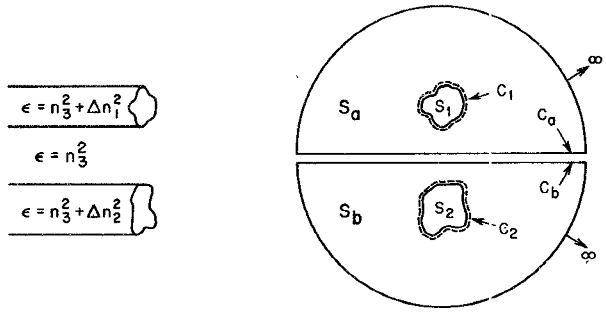

Fig. 1. Geometry of two parallel guides.

$\Delta\epsilon_1 = \Delta n_1^2$ and $\Delta\epsilon_2 = \Delta n_2^2$ are "difference" permittivities which, against the background of $\epsilon_3 = n_3^2$, form the two dielectric surface waveguides. We consider a single surface-wave mode on each of the guides 1 and 2, with field distributions in the transverse (x–y) plane given by $\bar{E}_{1,2}$ and $\bar{H}_{1,2}$ and propagation constants β_1 and β_2. The t and z dependence is assumed to be $\exp[j(\omega t - \beta z)]$ and is dropped when not required. If the guides are sufficiently far apart, the fact that the fields of each guide mode are evanescent outside the guide and thus, small compared to the fields of the other guide in its vicinity leads us to search for system modes with propagation constant Γ whose fields we represent approximately as

$$\bar{E} = m_1\bar{E}_1 + m_2\bar{E}_2 \quad \bar{H} = m_1\bar{H}_1 + m_2\bar{H}_2$$

with the relative components m_1 and m_2 of modes 1 and 2, as yet, arbitrary. The present approach is related to that of [6], [7].

Harrington [8] has derived a variational formula for the propagation constant of a general uniform lossy inhomogeneous isotropic waveguide

$$\beta = \frac{\omega\int[\epsilon\bar{E}\cdot\bar{E}^+ - \mu\bar{H}\cdot\bar{H}^+]\,dS + j\int[\bar{E}^+\cdot\nabla_t\times\bar{H} + \bar{H}^+\cdot\nabla_t\times\bar{E}]\,dS}{\int[\bar{E}_t^+\times\bar{H}_t - \bar{E}_t\times\bar{H}_t^+]\cdot\bar{a}_z\,dS}. \quad (1)$$

Here the integrals are over the (infinite) cross section of the guide, ∇_t is the transverse delta operator, and $\bar{E}^+$ and $\bar{H}^+$ are the so-called "transpose" fields corresponding to a mode traveling with propagation constant $-\beta$, and are related in this case to $\bar{E}$ and $\bar{H}$ by

$$\bar{E}_t^+ = \bar{E}_t \quad E_z^+ = -E_z \quad \bar{H}_t^+ = -\bar{H}_t \quad H_z^+ = H_z. \quad (2)$$

We note that β_1, $\bar{E}_1$, and $\bar{H}_1$ or β_2, $\bar{E}_2$, and $\bar{H}_2$ will individually satisfy (1) if $\Delta\epsilon_2$ or $\Delta\epsilon_1$ is set equal to zero, respectively. To find the system mode propagation constants Γ, we insert our trial fields into (1) and find the stationary values. We obtain

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$$\Gamma = \frac{m_1^2 N_1 + 2m_1 m_2 K + m_2^2 N_2}{m_1^2 P_1 + 2m_1 m_2 L + m_2^2 P_2} \quad (3)$$

where

$$P_{1,2} = 2 \int [\bar{E}_{t_{1,2}} \times \bar{H}_{t_{1,2}}] \cdot \bar{a}_z \, dS \quad (4)$$

and

$$N_{1,2} = \beta_{1,2} P_{1,2} + d_{1,2}$$

$$d_{1,2} = \omega\epsilon_0 \int \Delta\epsilon_{2,1} \bar{E}_{1,2} \cdot \bar{E}_{1,2}^+ \, dS$$

$$\Gamma_\pm = \frac{\beta_1' + \beta_2' - (2LK/P_1P_2) \pm [(\beta_1' - \beta_2')^2 + (4/P_1P_2)(K - \beta_1'L)(K - \beta_2'L)]^{1/2}}{2(1 - L^2/P_1P_2)} \quad (10a)$$

$$K = \frac{1}{2}\Big\{\omega\epsilon_0 \int (\Delta\epsilon_1 + \Delta\epsilon_2)\bar{E}_1 \cdot \bar{E}_2^+ \, dS - \beta_1 \int (\bar{H}_2^+ \cdot \bar{a}_z \times \bar{E}_1 + \bar{E}_2^+ \cdot \bar{a}_z \times \bar{H}_1) \, dS - \beta_2 \int (\bar{H}_1^+ \cdot \bar{a}_z \times \bar{E}_2 + \bar{E}_1^+ \cdot \bar{a}_z \times \bar{H}_2) \, dS\Big\}$$

$$L = \int [\bar{E}_1^+ \times \bar{H}_2 - \bar{E}_2 \times \bar{H}_1^+] \cdot \bar{a}_z \, dS.$$

All surface integrations are over the infinite cross section in the transverse plane; however, since the integrands for $d_{1,2}$ are zero outside $S_{2,1}$ (because $\Delta\epsilon_{2,1} \doteq 0$ outside $S_{2,1}$), these are only finite surface integrals.

Using the vector identity [2]

$$\int_A \nabla \cdot \bar{F} \, dS = \frac{\partial}{\partial z} \int_A \bar{F} \cdot \bar{a}_z \, dS + \oint_C \bar{F} \cdot \bar{a}_n \, dl \quad (5)$$

where A is an area in the transverse plane, C is its boundary, and $\bar{a}_z$ and $\bar{a}_n$ are unit vectors in the z direction and the outward normal direction to C, respectively, we let

$$\bar{F} = [\bar{E}_1^+ \times \bar{H}_2 - \bar{E}_2 \times \bar{H}_1^+] \exp [j(\beta_1 - \beta_2)z] \quad (6)$$

and after some manipulation obtain

$$L = \frac{c_1 - c_2}{\beta_1 - \beta_2} \quad K = \frac{\beta_1 c_1 - \beta_2 c_2}{\beta_1 - \beta_2} \quad (7)$$

where

$$c_{1,2} = \omega\epsilon_0 \int \Delta\epsilon_{1,2} \bar{E}_1 \cdot \bar{E}_2^+ \, dS. \quad (8)$$

Again, the infinite surface integrals for $c_{1,2}$ reduce to finite ones since the integrands vanish outside $S_{1,2}$. To determine the stationary values of Γ and the corresponding values of the only other unknown $q = m_2/m_1$, we apply the conditions $\partial\Gamma/\partial m_1 = 0$ and $\partial\Gamma/\partial m_2 = 0$, [6], [7]. The result is

$$q = \frac{(\beta_1' - \Gamma)P_1}{L\Gamma - K} = \frac{L\Gamma - K}{(\beta_2' - \Gamma)P_2} \quad (9a)$$

where

$$\beta_{1,2}' = \beta_{1,2} + d_{1,2}/P_{1,2} \simeq \beta_{1,2}. \quad (9b)$$

The term $d_{1,2}/P_{1,2}$ in (9b) is exponentially small to the second order with respect to $\beta_{1,2}$ for well-separated guides, because the exponentially decayed fields $\bar{E}_{1,2}$ are squared and only occur in the cross section of the other guide.

Solving (9a) for the propagation constant Γ, we obtain (10a) for the two possible system modes. Now since our separation assumptions further imply

$$L \ll P_{1,2} \quad \text{and} \quad K \ll \beta_{1,2} P_{1,2}$$

(exponentially to the first order), we may simplify the foregoing expression to yield

$$\Gamma_\pm = \beta_{av} \pm \Delta\Gamma \quad (10b)$$

where

$$\beta_{av} \simeq \tfrac{1}{2}(\beta_1 + \beta_2) \quad \Delta\Gamma \simeq (\Delta^2 + \delta^2)^{1/2} \quad (11)$$

and

$$\Delta = \tfrac{1}{2}(\beta_1 - \beta_2) \quad \delta^2 = c_1 c_2/P_1 P_2. \quad (12)$$

The ratio $q = m_2/m_1$ is also obtainable from (9) and (10a) or (10b) as

$$q_\pm \simeq (P_1/P_2)^{1/2} \left[\frac{-\Delta \pm (\Delta^2 + \delta^2)^{1/2}}{\delta}\right] \quad (13)$$

if we define δ from (12) as

$$\delta = \frac{c_2}{(P_1P_2)^{1/2}} (c_1/c_2)^{1/2} = \frac{c_1}{(P_1P_2)^{1/2}} (c_2/c_1)^{1/2}. \quad (14)$$

The two system-mode fields are thus

$$\bar{E}_{s\pm} = m_1(\bar{E}_1 + q_\pm \bar{E}_2) \quad (15)$$

and similarly for $\bar{H}_{s\pm}$. This means if we have a general field $\bar{E}$ given in terms of the coupling of the two individual waveguide modes as

$$\bar{E} = A_1(z)\bar{E}_1 + A_2(z)\bar{E}_2 \quad (16a)$$

this field must be uniquely expressible also in terms of the system modes

$$\bar{E} = A_+ \bar{E}_{s+} \exp(-j\Gamma_+ z) + A_- \bar{E}_{s-} \exp(-j\Gamma_- z) \quad (16b)$$

where A_+, A_- are mode amplitudes of the two system modes. We have, by comparing these two expressions, and the use of (15), the following relationships:

$$A_1(z) = A_+ \exp(-j\Gamma_+ z) + A_- \exp(-j\Gamma_- z)$$
$$A_2(z) = A_+ q_+ \exp(-j\Gamma_+ z) + A_- q_- \exp(-j\Gamma_- z). \quad (17)$$

In order to examine how the conventional coupled-mode equations can be derived from the present approach, we first solve from (17) for $A_\pm \exp(-j\Gamma_\pm z)$ and then differentiate with respect to z to obtain

$$j\frac{d}{dz}\begin{bmatrix} A_1(z) \\ A_2(z) \end{bmatrix} = \begin{bmatrix} \beta_1 & \delta(P_2/P_1)^{1/2} \\ \delta(P_1/P_2)^{1/2} & \beta_2 \end{bmatrix} \begin{bmatrix} A_1(z) \\ A_2(z) \end{bmatrix} \quad (18)$$

where use has been made of (10b) and (13). If we introduce normalized amplitudes $a_i = A_i(P_i)^{1/2}$ we can write the foregoing equation as

$$j\frac{d}{dz}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \beta_1 & \delta \\ \delta & \beta_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

which are the standard coupled-mode equations [1], [2], [4], [5] for this pair of modes. We note that our approach can, in fact, be generalized to include an arbitrary number of modes on an arbitrary number of waveguides, as well as to anisotropic media [9], whereas it is not clear how some of the other methods could be so generalized.

III. RELATIONS TO OTHER METHODS

The result (10b) derived here agrees with those obtained by Marcuse [1] and Snyder [2], but has the additional advantage of assuming second-order accuracy in the error fields when (10a) is used because of the variational nature of the technique employed. To investigate the relationships of this method to others in the literature, we consider the auxiliary geometries shown in Fig. 1. It will be noted that to calculate c_1 or c_2 requires a surface integration over the finite cross section S_1 or S_2, respectively. Arnaud [3] also obtains (11) and (12), with the exception that his coupling constants are given by contour integrals of the type

$$e_i = \oint_{C_i} [\bar{E}_1^+ \times \bar{H}_2 - \bar{E}_2 \times \bar{H}_1^+] \cdot \bar{a}_{ni}\, dl, \quad \text{for} \quad i = a,b \quad (19)$$

and $\bar{a}_{na}, \bar{a}_{nb}$ are outward unit normal vectors on contours C_a and C_b as indicated in Fig. 1. To transform (8) into a similar integral, we apply again the identity (5) to the vector $\bar{F}$ of (6), with A and C now replaced by S_a and C_a or S_b and C_b, and obtain

$$\begin{cases} c_1 = (\beta_1 - \beta_2)D_a - je_a \\ -c_2 = (\beta_1 - \beta_2)D_b - je_b \end{cases} \quad (20)$$

where

$$D_{a,b} = \int_{S_{a,b}} [\bar{E}_1^+ \times \bar{H}_2 - \bar{E}_2 \times \bar{H}_1^+] \cdot a_z\, dS.$$

Equation (20) holds for arbitrary contours $C_{a,b}$ and enclosed surfaces $S_{a,b}$ subject only to the condition that S_1 (respectively, S_2) be completely contained in S_a (respectively, S_b) (see Fig. 1). In the degenerate case ($\beta_1 = \beta_2$), and in particular when the finite portions of C_a and C_b are allowed to coincide we have $e_a = -e_b = jc_1 = jc_2$ and our result therefore coincides with that of Arnaud [3]. However, for nondegenerate modes, his results correspond to ours only if the $D_{a,b}$ terms can be neglected. We will see in the next section that the error incurred by this neglect may in fact be quite large if C_a is chosen close to one of the guides.

Alternatively, we may choose C_a and C_b to coincide with the boundary contours C_1 and C_2 of the individual guides. Now the $D_{a,b}$ become $D_{1,2}$ which are likely to be much smaller than in the previous case, and although we now have two contour integrals e_1 and e_2 to evaluate, they are both along finite contours and therefore presumably easier to calculate numerically. These coupling constants are the ones obtained in the detailed treatment of the lossless-fiber case by Vanclooster and Phariseau [4] whose connection with the Marcuse–Snyder form is mentioned by Snyder [2]. Again, $c_1 = c_2 = -je_1 = +je_2$ holds exactly in the degenerate case; all results are identical and the contour(s) may be taken anywhere outside the guides that suits the particular problem's geometry.

Jones [5] obtains the same form for the coupling constants as in [4] by a method which is noteworthy insofar as it is the only formally exact treatment, including the continuous-mode spectrum of both guides.[1] Although both [4] and [5] consider only the specific case of a circular fiber, it is evident from the derivation of (19) and (20) that their results apply to the general case. Matsuhara and Kumagai [6], [7] have used E- and H-field variational principles in a derivation similar to that used here with a mixed-field principle. These can be shown to give results close to ours in the present case if $|\Delta\epsilon_{1,2}| \ll |\epsilon_3|$ (i.e., the guides are weakly guiding) and the modes are degenerate [9], but in general, since these methods serve to determine β^2 rather than β, direct comparison with coupled-mode theory is more difficult.

IV. RESULTS FOR SLAB WAVEGUIDES

In order to quantitatively compare the various results presented here, we consider two parallel slab waveguides of widths $d_1 = 2a_1$ and $d_2 = 2a_2$ separated by a width d,[2] as shown in Fig. 2. Guide 1, guide 2, and the substrates are taken to have dielectric constants $\epsilon_1 = \epsilon_3 + \Delta\epsilon_1, \epsilon_2 = \epsilon_3 + \Delta\epsilon_2$, and ϵ_3, respectively, which are assumed to be constant scalars, but may be complex. For simplicity we consider only the coupling between even TE modes of these structures; results will be similar for other cases.

[1] Snyder's method [2], while exact for the isolated perturbed fiber, can only be heuristically extended to the multiple fiber case.

[2] No confusion of the widths d used above should arise with the quantities d_1 and d_2 used in Section II.

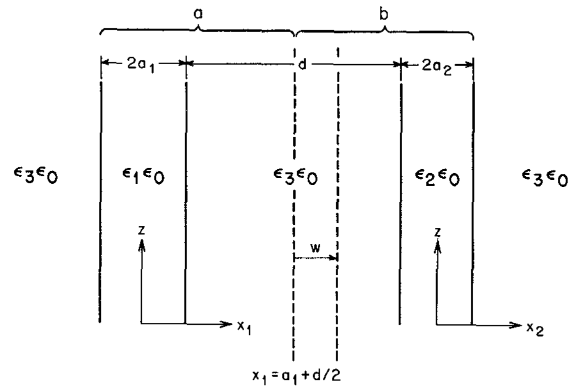

Fig. 2. Geometry of two parallel slab guides.

As is well known, we may write the fields of either guide in isolation as

$$\begin{cases} E_{y_i} = A_i \cos p_i x_i \\ H_{z_i} = -(p_i/\omega\mu_0) A_i \sin p_i x_i, \qquad |x_i| < a_i \\ H_{x_i} = -(\beta_i/\omega\mu_0) A_i \cos p_i x_i \end{cases}$$

inside the slabs, where $p_i^2 = k_0^2\epsilon_i - \beta_i^2$, and

$$\begin{cases} E_{y_i} = A_i \cos p_i a_i \exp\left[-\gamma_i(|x_i| - a_i)\right] \\ H_{z_i} = \dfrac{x_i}{|x_i|} \dfrac{j\gamma_i}{\omega\mu_0} A_i \cos p_i a_i \exp\left[-\gamma_i(|x_i| - a_i)\right], \qquad |x_i| > a_i \\ H_{x_i} = -\dfrac{\beta_i}{\omega\mu_0} A_i \cos p_i a_i \exp\left[-\gamma_i(|x_i| - a_i)\right] \end{cases}$$

outside the slabs, where $\gamma_i^2 = \beta_i^2 - k_0^2\epsilon_3$, and the β_i satisfy the characteristic equation

$$\tan p_i a_i = \gamma_i / P_i$$

or equivalently

$$\cos p_i a_i = \frac{p_i}{k_0(\epsilon_i - \epsilon_3)^{1/2}} \qquad \sin p_i a_i = \frac{\gamma_i}{k_0(\epsilon_i - \epsilon_3)^{1/2}}.$$

The previous equations are used to calculate the various quantities relevant to the coupling problem.

To compare the coupling coefficients of Marcuse, Snyder, Arnaud, and Jones for various degrees of nondegeneracy (in the degenerate case all three are equivalent as discussed in Section III), we calculate δ as used in (10)–(14), labeling them δ_M, δ_A, and δ_J, respectively, where

$$Q = \frac{p_1^2 p_2^2 \exp\left[-(\gamma_1 + \gamma_2)d\right]}{4\beta_1\beta_2(a_1 + 1/\gamma_1)(a_2 + 1(\gamma_2)}. \tag{24}$$

Also, note that our first-order result (10b) coincides with (21), but retaining all the terms in (10a) allows us to obtain a second-order approximation for $\Gamma_\pm$ as well. In Figs. 3 and 4 the differences between δ_M^2, δ_A^2, and δ_J^2 are plotted against the degree of degeneracy of the modes, due to differences between a_1 and a_2. These relative differences are independent of d, and become most pronounced

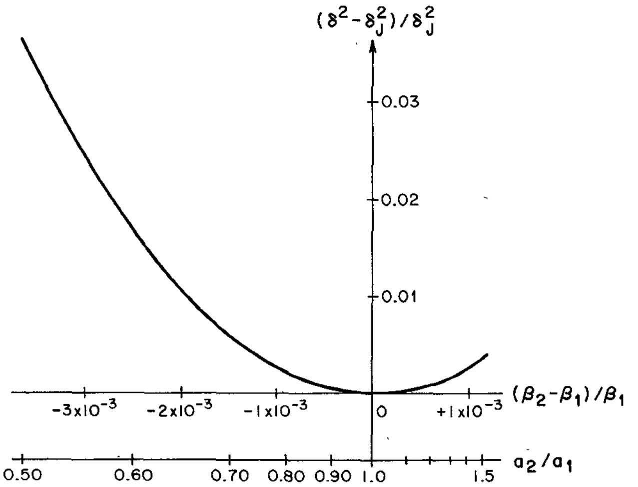

Fig. 3. Comparison of δ_J^2 and δ_M^2 for the TE_0 mode as a function of relative slab widths. $\epsilon_3 = 1.00$, $\epsilon_1 = \epsilon_2 = 1.04$, $k_0a_1 = 8.68$.

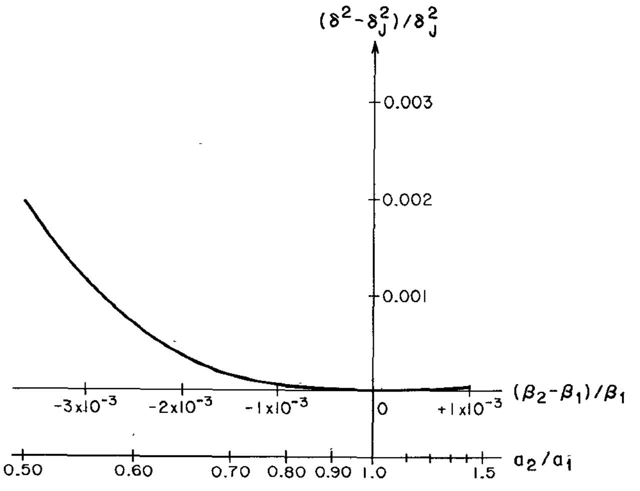

Fig. 4. Comparison of δ_J^2 and δ_A^2 for the TE_0 mode as a function of relative slab widths. $\epsilon_3 = 1.00$, $\epsilon_1 = \epsilon_2 = 1.04$, $k_0a_1 = 8.68$.

$$\delta_M^2 = Q\frac{\left[(\gamma_1 + \gamma_2) + (\gamma_1 - \gamma_2)\exp(-2\gamma_2 a_1)\right]\left[(\gamma_1 + \gamma_2) + (\gamma_2 - \gamma_1)\exp(-2\gamma_1 a_2)\right]}{\left[k_0^2(\epsilon_1 - \epsilon_3) + \beta_2^2 - \beta_1^2\right]\left[k_0^2(\epsilon_2 - \epsilon_3) + \beta_1^2 - \beta_2^2\right]} \tag{21}$$

$$\delta_A^2 = Q\frac{(\gamma_1 + \gamma_2)^2}{k_0^4(\epsilon_1 - \epsilon_3)(\epsilon_2 - \epsilon_3)} \tag{22}$$

$$\delta_J^2 = Q\frac{\left[(\gamma_1 + \gamma_2) + (\gamma_1 - \gamma_2)\exp(-2\gamma_2 a_1)\right]\left[(\gamma_1 + \gamma_2) + (\gamma_2 - \gamma_1)\exp(-2\gamma_1 a_2)\right]}{k_0^4(\epsilon_1 - \epsilon_3)(\epsilon_2 - \epsilon_3)} \tag{23}$$

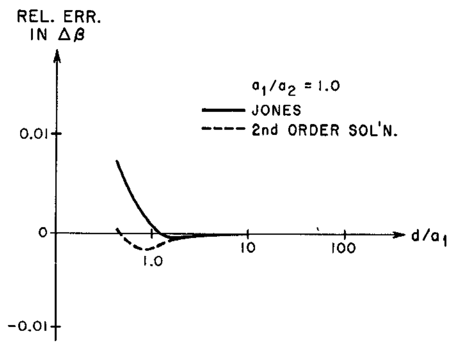

Fig. 5. Comparison of calculated exact value of $\Delta\Gamma = \frac{1}{2}(\Gamma_+ - \Gamma_-)$ for two degenerate TE_0 slab modes with Jones' value and the second-order value from (10). $\epsilon_3 = 1.00, \epsilon_1 = \epsilon_2 = 1.04, k_0a_1 = 8.68$.

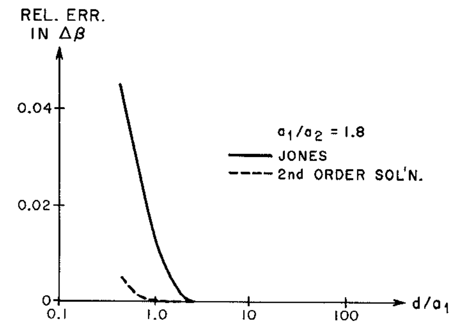

Fig. 6. Comparison of calculated exact value of $\Delta\Gamma = \frac{1}{2}(\Gamma_+ - \Gamma_-)$ for two nondegenerate TE_0 slab modes to Jones' value and the second-order value from (10). $\epsilon_3 = 1.00, \epsilon_1 = \epsilon_2 = 1.04, k_0a_1 = 8.68$.

when one of the modes comes close to cutoff, but are zero in the degenerate case.

In Figs. 5 and 6, $\Delta\Gamma = \frac{1}{2}(\Gamma_+ - \Gamma_-)$ is compared for a degenerate and a nondegenerate case of the result using Jones' coupling constant, the second-order variational result obtained by keeping all terms in (10), and the exact result, obtained by solving the exact modal characteristic equation for the system [10].[3] The variational result is quite superior close to the guide, when the other result becomes much more in error.

In Arnaud's paper [3] it was assumed that the finite part of the contour (which in the two-dimensional slab case degenerates from a line integral into simply the value of the fields at a single point: $x_1 = a_1 + d/2 + w$ in Fig. 2) could be chosen anywhere; however, in the nondegenerate case, it is apparent that this can result in a relative error of a factor as large as $\exp[\pm(\gamma_2 - \gamma_1)d/2]$ in evaluating δ_A. Now this is absorbed into a factor $\exp[-(\gamma_1 + \gamma_2)d/2]$; then, to assure an error bound on $\log \delta$ we require

$$|\gamma_1 - \gamma_2| \ll |\gamma_1 + \gamma_2| \tag{25}$$

or equivalently

$$|\beta_1 - \beta_2| \ll 4\frac{\beta_{1,2}^2 - k_0^2\epsilon_3}{\beta_1 + \beta_2}. \tag{26}$$

One consequence of (25) is that the closer one of the modes is to cutoff, the less nondegeneracy may be tolerated between the two modes to allow Arnaud's contour to be arbitrarily chosen.

Further, in order to be able to neglect the $D_{a,b}$ or $D_{1,2}$ terms which are the difference between Arnaud's, Jones', Marcuse's, and Snyder's coupling coefficients, we must be able to say

$$|\gamma_1 - \gamma_2| \exp(-2a_{1,2}\gamma_{2,1}) \ll |\gamma_1 + \gamma_2| \tag{27}$$

as well as

$$|\beta_1^2 - \beta_2^2| \ll k_0^2 |\Delta\epsilon_{1,2}|. \tag{28}$$

However, (25) clearly implies (27), and (26) similarly implies (28). Hence, the choice between any of these formulas is arbitrary so long as the approximate degeneracy condition (25), (26) holds. Moreover, if $|\Delta| \gg |\delta|$, the calculation of $\Delta\Gamma$ is virtually independent of δ, and since no significant power transfer occurs, this case is academic anyway.

V. CONCLUSION

It seems appropriate to conclude with a few general remarks about the assumptions necessary to treat the current problem by coupled-mode theory. In the first place, it is important to realize the sensitivity of any of the expressions for the coupling coefficients to the use of inexact fields therein. Marcatili [11], for instance, has obtained good results for the propagation constant of a rectangular dielectric waveguide sufficiently far from cutoff, even though the assumed fields outside the guide differ rather seriously from the actual ones. However, it has been shown [12], [13] that since the coupling coefficients are obviously quite strongly dependent upon the exterior field forms, the values of the coupling coefficient found in [11] can be quite inaccurate.

If it is desired to include more guided modes of the isolated guides in the analysis, the present variational technique can be extended in a straightforward fashion [9]. In general, to investigate coupling lengths and power transfer from a given mode, it is necessary to include all modes with values of β_i sufficiently close to the mode of interest; more quantitative criteria can be found elsewhere [2], [9], [14]. The coupling lengths and transferred powers will depend only upon the various system-mode propagation constants Γ_i (as well as the unperturbed β_i) which are calculated using a variational formula, and so will be relatively insensitive to the presence or absence of the fields of those modes with β sufficiently far from the β_i of the modes of interest. In Section IV it was seen that the $\Gamma_\pm$ were most accurately calculated for the slab case using (10), with the second-order terms retained. This, as well as retaining additional modes, will give a higher degree of accuracy in the coupled-mode analysis of such systems.

Strictly speaking, we should include continuous spectrum (radiation) modes to accurately represent the fields of the system. Since each guide possesses a set of such modes which is (in conjunction with the surface-wave modes) complete, a certain arbitrariness is inherent in the expansion of system-mode fields in such a case. The vari-

[3] Clearly, for such a highly nondegenerate case as depicted in Fig. 6, the coupling will be poorly described by only two modes, and more would be required to represent the situation with reasonable accuracy (see Section V).

ational method automatically adjusts the amplitudes of surface-wave modes in the appropriate manner to construct the system modes, but the various integrals in Section II will diverge if radiation modes are used, so that some modification of the present method would be necessary to treat problems of this type.

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Single-Mode Pulse Dispersion in Optical Waveguides

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Abstract—**The limitations of a widely used method for analyzing pulse distortion in a single-mode waveguiding structure are derived. The results are applied to propagation in optical waveguides, and for cases where material dispersion is dominated by a broad resonance line, pulse attenuation is found to be much more serious than the broadening of the pulse. In extremely low-loss regions, however, other effects may cause the reverse to be true.**

I. INTRODUCTION

WITH THE RECENT development of extremely low-loss optical waveguides [1] making feasible long-distance transmission via this medium, there has been increased interest in determining the pulse characteristics of such devices [2], [3]. These characteristics are determined by the nonlinearity of the $\beta - \omega$ characteristics of individual modes, and, in multimode guides, by the differences in group velocity between different modes. In this paper we address ourselves to the first of these causes, referring the reader to [3] for a discussion of the second.

We shall obtain a more precise formulation for the region of validity of a widely used technique [2], [4]–[6] for analyzing, approximately, the distortion of a single pulse, and apply these results to the study of such pulses in optical waveguides.

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II. PROPAGATION OF A GAUSSIAN PULSE

We seek to analyze the behavior of a pulse of Gaussian envelope

$$f(t) = (a\sqrt{\pi})^{-1/2} \exp(-t^2/2a^2) \exp(j\omega_0 t) \qquad (1)$$

with center frequency ω_0 and width a which has been normalized to unit strength

$$\int_{-\infty}^{\infty} |f(t)|^2 \, dt = 1$$

as it propagates along an arbitrary transmission channel of transfer function $S(\omega) = \exp[-j\beta(\omega)L]$. Here L is the length of a section of the channel between the input ($z = 0$) and the output, and $\beta(\omega) = h(\omega) - \frac{1}{2}j\alpha(\omega)$ is the frequency-dependent propagation constant of the channel split into phase constant and (power) attenuation constant. The output signal is then represented by the usual Fourier-transform method

$$q(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) S(\omega) \exp(j\omega t) \, d\omega \qquad (2)$$